AD-757 708

An Earth Heat Sink Concept for Underground Power Sources

Naval Civil Engineering Laboratory

FEBRUARY 1973

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AD757708

Technical Note N-1261

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By

S. C. Garg, Ph. D.

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Approved for public release; distribution unlimited

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Civil Engineering Laboratory Port Hueneme, California 93043	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

AN EARTH HEAT SINK CONCEPT FOR UNDERGROUND POWER SOURCES

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

5. AUTHOR(S) (First name, middle initial, last name)

S. C. Garg, Ph. D.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
February 1973	36	42

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. Z-R000-01-139	TN N-1261
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Director of Naval Laboratories, Washington, D. C.

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DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6801

Unclassified

Security Classification

IIb

Unclassified

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Heat sinks						
Soils						
Underground facilities						
Cooling						
Electric power plants						

DD FORM 1 NOV 65 1473 (BACK)

(PAGE 2)

TLC

Unclassified

Security Classification

TABLE OF CONTENTS

INTRODUCTION

To ensure uninterrupted operation of critical underground power and communication installations during and after nuclear attack conditions, it is necessary that such facilities be isolated from or extensively hardened against overpressures and ground motion. The extent of hardening of such facilities will depend upon the design concept of the entire installation, expected overpressures, the extent of redundancy to assure facility survival, availability and feasibility of obtaining equipments which can withstand attack conditions, costs, and a number of other operational considerations. In any extensive hardening, it is necessary to consider the possibility of completely sealing off the entire installation from dependence on any surface facility. A major consideration associated with such isolation is the problem of dissipation of the power system's waste heat. The requirement for a large heat sink becomes obvious for any major installation.

The common method of heat rejection during preattack operation of such facilities would probably be to the surface above ground using air or water as the medium. This is a necessary requirement for facilities which are expected to be operating most of the time during peacetime. However, such installations may be required to function in a closed cycle during and after attack conditions. This requirement makes it necessary that the waste heat be stored underground for a specified period of time.

Several alternative methods of heat storage for underground facilities have been considered in prior studies for conditions where the heat storage systems were located so deep in the ground that they were not expected to be subjected to significant overpressures or ground motion during attack. For such systems, stored chilled water, ice-water mixtures, and solutions of various chemicals in water were analyzed as possible heat sinks [1]. Detailed analyses of these heat sinks and a consideration of rock heat sinks have been carried out by various U. S. Governmental agencies, some of which are listed as references [2-13] in this report. The power source considered in these analyses was a nuclear reactor of capacity in the megawatt range. A later experimental study [14] reports the results of a test program carried out to determine the feasibility of a rock heat sink. The results showed severe structural failures due to thermal stresses and moisture penetration in the rocks.

For facilities which require power in the order of 100 to 500 kw and which are located closer to the surface so that they are subjected to the overpressures and ground motion during attack, the design considerations are entirely different from those considered in studies cited above. For such applications, although storage of ice or an ice-water mixture is possible, it may be very costly if the specified period of post attack operation is more than a few days. Besides cost, the ice or ice-water mixture heat sinks would also require regular or periodic surface support to remove the heat gained from surrounding structures or ground during the standby period. Another possible heat sink could be the presence of aquifers in the vicinity of the installation. To use aquifer as a heat sink, shock-isolated wells would have to be drilled for withdrawal and reinjection of water. The parameters for aquifer development and use are highly site-dependent, requiring extensive geological determination and evaluation at each proposed site [15-18].

Another method of underground storage of heat is the use of earth as a heat sink. If the temperature of the soil surrounding an underground installation could be raised, it can act as an excellent heat sink. To accomplish this, excavation of the ground would have to be carried out to permit installation of pipe grids followed by burying the grids in select backfill of high thermal conductivity and specific heat. The spacing of pipes in the grid, pipe sizes, dimensions of the earth heat sink including allowances around it to reduce the effects of ground motion to an acceptable minimum, the amount of moisture in the backfill, and the density, particle size and type of backfill are among factors which will require careful design and evaluation.

This report begins by summarizing the efforts of a literature survey carried out to gather available information on soils and their properties which determine the volume of soil that will be required to absorb a predetermined amount of heat over a specified period of time. To ascertain the practicality of earth heat sinks for such applications, preliminary calculations were then carried out to determine the order of magnitude of dimensions of the required heat sink. These calculations have shown that earth heat sinks may be practical and that they should be considered along with other methods in the selection process for a heat dissipation medium for installations operating on a closed cycle basis over limited durations in time.

GENERAL CONSIDERATIONS

The use of earth as a heat sink or source requires very different methods of analysis from those used when the atmosphere or a large body of liquid is used. For air, water or ice heat sinks, the heat transfer does not produce temperature gradients in the medium because of convection; hence, analytical methods involving steady state analyses are adequate. When heat is added to or extracted from earth, however, heat diffusion occurs so slowly that marked temperature gradients are established which vary as a function of time. For such earth embedded heat transfer surfaces, the design must be in terms of transient heat flow, requiring transient heat conduction analyses. The importance of transient analysis is brought out by the fact that variations in heat transfer rates with time are of the order of thousands of percent. For example, if a large flat plate buried in earth is raised suddenly by 35° F above the steady state earth temperature and maintained at that level, the rate of heat loss from the flat plate after 1, 16 and 3600 hours will be approximately 20, 5 and 1/3 Btu per hour per square foot [19], respectively, for a soil of diffusivity of 0.025 square ft/hr. Thus, it is very evident that any statement as to the heat transfer rate from buried surfaces is meaningful only if the heat transfer rate is an instantaneous one applicable after a specific number of hours subsequent to initial operation.

Any underground power source which is used only under attack or post attack conditions, when the peacetime surface power sources are out of operation, may be designed to be used only once, and that too for a limited duration in time of perhaps not more than two weeks or so. For such a short duration, compared to the use of earth as a heat source or sink for year-round operation in a heat pump cycle for home conditioning, the heat transfer rates from buried pipes may be reasonably high at the end of the designed operating period to offer earth as a useful alternative heat sink for underground power sources. This may be especially true if care is taken in selecting a soil of high thermal conductivity and specific heat.

Beyond depths of just a few feet, the soil temperature at any location has a steady year-round value which remains largely unaffected by seasonal climatic changes on the surface. The steady state soil temperature at a depth of 30-60 feet may vary anywhere from 37°F to 72°F in the United States, depending upon the location [20]. Below a depth of about 30 feet, the steady earth temperature further increases by about 1°F for each 60 feet increase in depth. Therefore, for all practical purposes, the earth temperature at a given location may be assumed to have a constant value before the heat exchanger embedded in the ground has been used either to extract heat from or to add heat to it.

REVIEW OF RELATED WORK

During the 1940's and early 1950's, earth was seriously considered as a possible source of storage medium for heat in the United States. Investigations were carried out into the feasibility and economics of using earth in a heat pump cycle for year-round air conditioning of homes. As heat pump cycles turned out to be more expensive than gas or oil heating systems, interest in them subsided causing a virtual halt of investigations in this area. Most of the published work in heat pump investigations is, therefore, 20-30 years old. In the following survey, the properties of soils which determine the transient heat transfer rates will first be discussed, followed by the analytical considerations and test results as presented in the literature.

Soil Properties

The soil properties which determine the rate of heat transfer between an embedded pipe and surrounding earth are density, porosity, fluid permeability, moisture contents, thermal conductivity, specific heat, thermal diffusivity and particle size. Properties like specific heat and moisture contents are determined by the chemical and mineralogical composition of the soil, water tables and porosity. Properties like thermal conductivity, porosity and permeability are strongly dependent upon the form of rock, particle size and packing arrangement. All of these properties affect heat transfer rates through their affect upon the thermal diffusivity of the soil.

The constituents of soil vary from place to place as do particle sizes and distribution, density, porosity and the amount of moisture present. It is, therefore, obvious that the thermal diffusivity of soils will vary widely from one location to another. To provide some idea of these variations, the various types of rocks subjected to thermal conductivity determination in one investigation may be cited [21-22]. Some of the rock samples studied in this effort were: granite, monzonite, tonalite, augite, syenite, albitite, anorthosite, diabase, gabbro, hypersthenite, bronzitite, dunite, sandstone, marble, gneiss, slate, limestone, dolomite, calcite, halite, and quartz. Each of these samples contained varying percentages of other minerals, and the crystal size varied between 0.001 and 3.0 millimeters. The thermal conductivity of these unfractured dry rocks was found to vary between 1.0 and 2.7 Btu/hr-ft-^{o}F. The density variation was, however, smaller since these materials were in solid, dry form. In powdered form, with particles of size between 10 and 500μ, the density of inorganic soils varies between 60 and 120 lbs/cu ft. Wide variations in soil composition in one state, and significant variations with depth at individual sample locations have also been reported [23]. Some other published values of thermal conductivity of unfractured rocks are: 0.7 to 1.7; Btu/hr-ft-^{o}F, [24], 1.9 Btu/hr-ft^{o}F, [25], and 1.52 to 3.46 Btu/hr-ft-^{o}F, [26]. A summary of other published values of the thermal conductivity of solid rocks ranging from 0.6 to 3.5 Btu/hr-ft-^{o}F is given in reference [27].

For the purpose of earth heat sink applications, however, the important thermal conductivity and diffusivity values are those of soils in powdered form containing varying percentages of various minerals and water. If the water tables are high, the percentage of moisture will depend largely upon the porosity of soil. A powder of uniform size solid spheres would have a porosity between 25.95% (rhombohedral packing arrangement) and 47.64% (cubic packing arrangement). Since soil particles at any location are likely to consist of irregular grain shapes of sizes varying over a wide range, a much larger variation in porosity should be expected. Furthermore, a loosely sifted powder will have a much larger porosity than one which has been compacted by mechanical means. Porosity variations for powder and granular materials have been found to vary from 33-50% for Olivine Basalt Powder to 63-75% for Chondrite powder [27] .

Several experimental investigations to determine the thermal conductivity of soils containing known amounts of moisture have been reported in the literature [28-30] . Smith and Yamauchi [28] tested five different soils, each with several different moisture concentrations. Each soil was first examined to determine the distribution of particle sizes. Of five samples, three were classed as Sandy Type Soils with grain sizes of between 0.08 and 4.0 millimeter for 70% of the soil. The other two soils were Clay and Loam with grain sizes of between 0.005 and 0.08 millimeter for 80% of the soil. The moisture contents as a percentage of dry weight, densities and thermal conductivities at room temperature for the two types of samples are given in Table 1 to provide some idea of the effects of moisture and density upon the thermal conductivity. Detailed curves are provided in the original publication which show strong dependence of thermal conductivity of a particular type of soil upon both the percentage of moisture present in the soil and the density of the soil. A more detailed investigation of thermal conductivities of soils carried out for the U. S. Army Corps of Engineers is available [29]. Properties of 13 different soils, each with several different moisture concentrations, are summarized in reference 30.

An examination of published literature suggests that the thermal conductivity of soil may vary between around 0.3 and 1.5 Btu/hr-ft-^{o}F, depending upon the composition of the soil, grain size and the amount of moisture. If select backfills were used for the purpose of providing earth heat sinks, it is quite easy to select soils with high thermal conductivities and specific heats. However, the percentage of moisture will depend to some extent upon the surrounding ground or structure. If the ground is relatively dry, water from the moist, select backfill will diffuse into it with time thereby reducing the effectiveness of the heat sink. For the purpose of preliminary calculations to determine the order of magnitude of the heat sink dimensions, a conservative thermal conductivity value of 0.5 Btu/hr-ft-^{o}F may be assumed.

The other two properties of the soil which determine heat transfer rates under transient conditions are the density and the specific heat of the soil. The density of inorganic soils vary between 60 and 120 lbs/cu ft, depending upon their composition. For the purpose of preliminary analysis, a soil density of 90 lbs/cu ft appears reasonable.

The specific heat of the soil will depend strongly upon the percentage of moisture present since the specific heat of water at room temperature of 1.0 Btu/lb-^{o}F is about four to five times that of rocks. For moist soils, values of the specific heat considered in the literature [19, 31, 32] range between 0.20 and 0.45 Btu/lb-^{o}F. A reasonable value for this perliminary analysis may, therefore, be assumed to be 0.3 Btu/lb-^{o}F.

With assumed values of the thermal conductivity of 0.5 Btu/hr-ft-^{o}F, density of 90 lbs/cu ft and specific heat of 0.3 Btu/lb-^{o}F, the value of soil thermal diffusivity for sample calculations is 0.0185 square ft/hr.

Analytical Considerations

Consider an infinitely long permanent line source or sink for heat in an infinite soil medium at an initial uniform termperature, i.e., constant heat flux at the pipe wall. Subsequent temperatures at any point in the medium may be represented by [33,34]

$$T = T_o + \frac{Q'}{2\pi k} \int_X^\infty \frac{e^{-\beta^2}}{\beta} d\beta \qquad (1)$$

where T_o = Initial uniform temperature of soil, ^{o}F.

T = Temperature of soil at time t and distance r, ^{o}F.

Q' = Heat Transfer rate across the pipe, Btu/hr per foot length.

k = Thermal conductivity of soil, Btu/hr-ft-^{o}F.

$X = \frac{r}{2\sqrt{\alpha t}}$.

r = Distance from the center of pipe, ft.

α = Thermal diffusivity of soil, $k/\rho C_p$, sq ft/hr.

ρ = Density of soil, lb/cu ft.

C_p = Specific heat of soil, Btu/lb-^{o}F

t = Time from start of operation, hr.

β = Variable of integration.

Ingersoll and Plass [32] offered Equation (1) in the form

$$\Delta T = \frac{Q'}{2\pi k} I(X)$$

where

$$I(X) = \int_X^\infty \frac{e^{-\beta^2}}{\beta} d\beta$$

and provided values of the integral I(X) for values of X ranging between 0.0001 and 2.20 in tabular form.

Although Equation (2) is exact only for a true line source, it may be applied with negligible error, after a few hours of operation, to small diameter pipes in actual use in heat pump installations. In fact, for Fourier numbers (N_F) of ≤ 0.05, the error [35] in using Equation (2) is less than 2%. For values of the Fourier number ($r^2/\alpha t$) larger than 0.05, a more general equation which is applicable for all values of the Fourier number may be used. This general equation for a constant heat transfer rate may be written in the form [36] ,

$$T - T_o = - \frac{Q'}{k\pi^2} \int_o^\infty (1-e^{-\beta^2 Z}) \left[\frac{J_o(p\beta)Y_1(\beta) - J_1(\beta)Y_o(p\beta)}{J_1^2(\beta) + Y_1^2(\beta)} \right] \frac{d\beta}{\beta^2} \quad (3)$$

Where $Z = \frac{1}{N_F} = \frac{\alpha t}{r^2}$

$p = r/r_1$

r_1 = Radius of the pipe

J_o, J_1 = Bessel functions of the first kind of order zero and one, repsectively.

Y_o, Y_1 = Bessel functions of the second kind of order zero and one, respectively.

For earth heat sinks for underground applications, the pipe length is going to be several hundred times the pipe diameter. Furthermore, as shown later, the values of Z are going to be much larger than 20. Therefore, Equation (2) may be adequate for applications using single pipes buried in the ground.

It is very unlikely, however, that a single pipe will be adequate to handle heat loads of more than just a few thousand Btu's per hour, unless the pipe is made several miles long. From a realistic and cost effective viewpoint, however, it will be necessary to use several "hairpin loop' pipes in parallel. Ingersoll and Plass [32] considered two parallel pipes of radius r_1, r_2 feet apart, in a hairpin loop, and proposed

$$T_1 - T_o = \frac{Q'}{2\pi k}\left[I(X_1) + I(X_2) \right] \tag{4}$$

Where $X_1 = r_1/2\sqrt{\alpha t}$

$X_2 = r_2/2\sqrt{\alpha t}$

T_1 = Temperature at pipe wall of radius r_1.

For an actual ear .. heat exchanger, practical considerations would dictate the use of long parallel pipes in a 3-dimensional array. For such an application, there is no reason why Equation (4) cannot be mofidied to

$$T_1 - T_o = \frac{Q'}{2\pi k} \sum_{n=1}^{\infty} I(X_n) \tag{5}$$

Where $X_n = r_n/2\sqrt{\alpha t}$

and r_2, r_3, r_4, etc., are distances of surrounding pipes in the array from the center of the pipe of radius r_1 under consideration.

A somewhat different equation will result if instead of maintaining a constant heat flux on the surface of the pipe, a constant pipe wall temperature was maintained. For a buried pipe whose temperature is raised above or lowered below the uniform earth temperature and maintained at that level, the heat transfer rate will decrease asymptotically with time. Equation for this constant pipe temperature case may be written in the form [37] :

$$Q' = \frac{8K\Delta T}{\pi} \int_0^{\infty} \frac{e^{-\beta^2 Z}}{J_o^2(\beta) + Y_o^2(\beta)} \cdot \frac{d\beta}{\beta} \tag{6}$$

This equation has been evaluated by Ingersoll, et.al. [35] in the form of tabulated values of F(Z), where

$$F(Z) = \frac{8}{\pi} \int_0^{\infty} \frac{e^{-\beta^2 Z}}{J_o^2(\beta) + Y_o^2(\beta)} \frac{d\beta}{\beta} \tag{7}$$

for values of Z ranging between 0.01 and 25,000.

A constant heat transfer rate from the surface of a very long cylinder (thereby eliminating end effects) embedded vertically in soil was considered by Guernsey, Betz and Skau [31] . Earth of infinite extent at a uniform temperature was assumed and the effects of heat interchange with atmosphere was neglected. With these assumptions, they obtained an approximate solution,

$$T - T_o = \frac{Q'}{5.4575k}\left[\log_{10} Z + 0.78/Z + 0.351\right], \tag{8}$$

with an error of less than 1% for $Z > 6$. Using this equation, they computed the lengths of various diameter pipes required to extract heat at a fixed rate over a 20-year period for a maximum predetermined value of $(T - T_o)$. They found that increasing the pipe size from 3/4-inch to 3.82-inch, a 5.1-fold increase, decreases the required length by only 20%. It therefore appears that smaller diameter pipes are more effective. This conclusion is reasonable since a longer, smaller diameter pipe is exposed to a larger volume of earth with which heat can be exchanged.

All the above equations ignore the effects of convection in the air or in the moisture present in the soil because it does not appear that convection currents in soils can be significant. Published geological studies indicate that convection in porous earth strata is extraordinarily slow [38] .

A somewhat different application has been analyzed by Kingston [39] which may be important in consideration of earth heat sinks for underground applications. He analyzed the cooling of mass concrete by water flowing in embedded pipes. This problem occurs in cooling of dams. He assumed that the water flowing through a pipe cools the concrete in a prism, the cross-section of which is a hexagon. The hexagon was approximated by a circle and it was assumed that the outer boundary of the infinitely long cylinder was thermally insulated. The entire concrete was assumed to be initially at a uniform temperature and subjected to a zero temperature at the surface of a cylindrical hole (equivalent to the pipe wall) concentric with the axis. The differential equation to be solved in this case was

$$\frac{\partial T}{\partial T} = \alpha\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] \tag{9}$$

with the boundary conditions

$$T = 0 \text{ at } r = r_i$$

$$\frac{dT}{dr} = 0 \text{ at } r = r_o$$

Where r_i = Inner radius of cylinder

r_o = Outer radius of cylinder

The solution of Equation (9) was quite involved and required determination by numerical methods.

Experimental Considerations

Smith [40] carried out an experimental investigation to determine the heat transfer rates as a function of time for a single isolated pipe, for two parallel pipes with varying spacings and for three parallel pipes. He compared his single pipe data with Equations (2) and (6) by plotting $Q'/k\Delta T$ against Z for both equations and the data, and found good agreement. The difference between the two equations was found to be small, and this difference was found to decrease with an increase in the value of Z. The heat coefficients and the temperature differences between pipe wall and initial earth temperature in the test using three pipes in parallel were found to agree very well with values predicted by Equation (5). The effects of the presence of other pipes upon the heat transfer rates from a pipe were presented in graphical form. The ratios of heat transfer rates from a pipe in arrays of of three and infinite number of pipes to its value for a single pipe were also plotted against Z for various spacings between pipes in the arrays.

Hadley [41] correlated all available experimental data from heat pump ground coil installations from eight different sites by plotting $K\Delta T/Q'$ against Z and found that

$$\frac{k\Delta T}{Q'} = 0.311 \log_{10} Z - 0.3696 \quad . \tag{10}$$

When he compared Equation (10) with Equation (2), he found that Equation (2) tends to predict conservative values for design purposes. For example, the experimentally obtained value of $Q'/k\Delta T$ from Equation (10) was about 2.25 times the value predicted by Equation (2) at Z = 100. At Z = 1000, the experimental value of $Q'/k\Delta T$ was approximately 30% higher than the predicted value. The difference approached zero as Z approached 20,000. From this comparison, it may be readily seen that Equation (2) should provide somewhat conservative designs.

THEORETICAL CONSIDERATIONS

The use of an earth heat sink to absorb large quantities of heat over a relatively short period of time requires a large surface area of the pipe, but a small soil volume because heat transmitted to soil at the pipe wall can diffuse only a short distance in the soil over the anticipated 2-3 weeks of operation. To minimize excavation and backfill costs, therefore, it seems reasonable to install long pipes in such a way as to enclose the required amount of soil in a parallelopiped arrangement. Either of the two pipe arrangements whose cross-sections are shown in Figures 1 and 2 is adequate for this application. For the purpose of thermal analysis, both of these arrangements could be approximated by a bank of hollow soil cylinders of inner and outer radii of r_i and r_o. The outer boundary of the cylinder at $r = r_o$ may then be assumed to be perfectly insulated so that no heat transfer can take place across its boundary. Although this assumption is incorrect for cylinders which form the boundaries of the matrix, their effect upon the total heat transfer capability of the matrix may be assumed to be negligible if the matrix has a large number of pipes and if the dimension r_o is such that the heat transfer rate from pipes of radius r_i after the specified time differs little from that of the pipe located alone in an infinite earth medium. Furthermore, if the pipes are long enough, the end effects may be assumed to be negligible. The hexagonal prism arrangement of Figure 2 provides a better approximation becuase it encloses only 9.31% of earth volume that will not be accounted for in this analysis compared with the unaccountable 21.46% in the Figure 1 arrangement. With these assumptions, the appropriate unsteady state heat transfer equation that has to be solved is the same as Equation (9), but with different initial and boundary conditions. To simplify analysis, the zero point of the scale can be shifted to the steady state initial earth temperature. The temperatures thus obtained from the solution of Equation (9) will represent the rise of temperature above the steady state earth temperature. The boundary conditions are, therefore,

$$T = T_1 \text{ at } r = r_i$$

$$\frac{dT}{dr} = 0 \quad \text{at } r = r_o$$

The solution of this equation involves Bessel functions of the first and second kind and of the first and second order. The solution is, therefore, quite involved, time consuming, and requires the extensive use of a high speed computer. This will be carried out for wide ranges of α, Z and t during the second phase of this investigation. For the purpose of this report, however, preliminary estimates of the sizes of earth heat sinks were carried out as described below using Equation (5).

PRELIMINARY CALCULATIONS

Design Criterion

Inexpensive power sources in the 100 kw to 500 kw range are non-nuclear in nature. Suitable non-nuclear power sources that might be considered for underground applications include Diesel, Gas Turbine and Fuel Cells. The combustion products or exhausts of Diesel power sources, for example, are at temperatures approaching 1000°F. Since the efficiency of these power sources does not depend upon the sink temperature, it should be possible to operate the heat sink at fairly high temperatures. However, because of the recycle nature of closed cycle operation, the operating temperature limit of the earth heat sink may be determined by the temperature of the processed exhaust which has to be recycled to the power source. For the purpose of this discussion, only a Diesel system will be considered.

In a closed cycle diesel operation, the inlet temperature of fuel, oxidizer and diluent (or heat absorber) has to be about 180°F if the engine is operating in a psychrocycle mode, to ensure the presence of sufficient moisture to prevent the cylinder temperature from rising beyond design limits. If the engine is to operate on a kreislauf cycle, however, the inlet temperature has to be somewhat higher than this value to ensure combustion. In either case, the engine cooling jacket or radiator temperature is limited to about 212°F when using ebullient cooling in commercially available units. The earth heat sink can therefore be subjected to a step temperature rise at pipe walls to at least 212°F, and possibly higher. One method of using the earth heat sink to absorb heat at 212°F will be to cool the combustion products (CO_2, O_2 and H_2O) from around 1000°F to around 212°F in a series of water cooled heat exchangers prior to the absorption of carbon dioxide. The steam thus formed in the radiator and the heat exchangers can be used to carry heat to the earth heat sink and returned to the radiator and the heat exchangers as liquid water at about 212°F. The pipe wall temperature may, therefore, be assumed to be subjected to 212°F over the entire duration of operation.

The actual situation in such a heat exchanger will be more complicated since, although the temperature of steam entering and that of water leaving the heat sink has been assumed to be at a constant value of 212°F, the heat output rate of power unit that has to be absorbed by the heat sink is also a constant. Therefore, the assumed initial and boundary conditions will cause a progressively larger length of the pipe to become active in the heat exchange process, as the heat transfer rate from the pipe section exposed to steam drops with time. The heat transfer rate from any given segment of the pipe will, therefore, vary with time as will its wall temperature as cool water is replaced by warmer water which is finally replaced by steam at 212°F. The upstream sections of the pipe will be the first to be exposed to 212°F steam whereas the downstream end will be exposed last at a time when although the entire heat exchanger upstream is being maintained at 212°F, the total heat transfer rate is less than the total waste heat output rate of the power unit. Similarly, the condensate temperature will probably start from slightly above the initial earth temperature, rising to 212°F at the end of the period.

Obviously, the constant heat flux assumption of Equation (5) is not applicable for any small section of the pipe, although it is applicable to the earth heat sink as a whole. The constant pipe wall temperature case may be somewhat more appropriate here, although the duration of exposure to steam will vary from 100% of engine operating period at entrance to the heat sink to only a fraction, as yet undefined, at the heat sink outlet. For several reasons, however, Equation (5) was selected for the preliminary calculations shown here: (a) the constant temperature case, Equation (6), cannot be utilized for a 3-dimensional matrix; (b) as shown by Smith [38] , there is only a small difference in heat transfer rates between the constant wall temperature and the constant heat transfer rate cases for values of Z of 500 or more; (c) Equation (5) can be used for a bank of pipes, as confirmed by experimental investigation of Smith [38] , and (d) the actual heat sink will experience conditions somewhere between the constant temperature and the constant heat transfer rate cases, for which equations are not available. To permit application of Equation (5), it was assumed that the entire pipe matrix is raised to 212°F at the start of the power unit. The heat transfer rate calculated at the end of the specified period was then assumed to be applicable to the whole pipe matrix. This assumption will obviously provide conservative values of the heat transfer rates since, after the specified period, the pipe sections downstream will have a higher heat transfer rate because they have not been exposed to 212°F for the entire test duration.

Heat Transfer Rate at End of Specified Period

From Equation (5), factors which determine the heat transfer rate at the end of the specified period are: thermal diffusivity and the thermal conductivity of soil, the radius and spacing of pipes, the pipe wall to initial earth temperature difference and the specified period of time.

Conservative values of thermal conductivity and thermal diffusivity of soil were earlier determined to be 0.5 Btu/hr-ft-°F and 0.0185 sq ft/hr, respectively. A futher assumption may be made that the soil is at a steady state temperature of 60°F at the start of its use as a heat sink. This assumption may again be somewhat conservative in view of the soil temperature variation in the United States of between 37° and 72°F. The value of ΔT is therefore assumed to be equal to 152°F.

Three time periods will be assumed for these calculations: one week, two weeks and three weeks. It is doubtful that any emergency underground system will be designed to operate for much less than one week or much more than three weeks. For the purpose of preliminary calculations, therefore, we have

$$k = 1.5 \text{ Btu/hr-ft-}^{\circ}\text{F}$$

$$\alpha = 0.0185 \text{ sq ft/hr},$$

$$\Delta T = 152^{\circ}\text{F},$$

and $$t = 168, 336 \text{ and } 504 \text{ hours}.$$

The last item we need in calculating heat transfer rates from the embedded pipes is the size and spacing of pipes. The pipe diameter should not be too small since it may cause a large pressure gradient in steam flowing through it, or too large since there is little heat transfer improvement [31] as compared to the costs. Furthermore, to reduce the pressure drop, several small diameter pipes may be connected in parallel through headers. Based upon these considerations, a pipe of outer diameter 1.5-inch was selected. For a pipe of radius 3/4-inch, the values of Z for the three selected time periods were 796, 1591 and 2387, respectively. For these values of Z, pipe spacings were determined, by trial and error, which caused the heat transfer rates to be approximately half those from a single pipe located in an infinite earth medium during the same period. Acceptance of some deterioration in the heat transfer rate was considered necessary to reduce the volume of the earth heat sink, and the 50% deterioration level was chosen arbitrarily. Under the same assumptions and design conditions, the pipe spacing will increase with an increase in the value of Z. In the hexagonal prism matrix of Figure 2, distances between the center of a pipe and the centers of six surrounding pipes, $2r_o$, were estimated to be 25-inches, 30-inches, and 40-inches for Z = 796, 1591 and 2387, respectively. Values of $I(X_n)$ were then obtained for the different pipe locations in the matrix followed by determination of the average heat transfer rate as follows.

As a first approximation, 24 rows of 24 pipes each were assumed in the matrix, a section of which is shown in Figure 2. A pipe in any location was then considered to be influenced by pipes no further than a distance of $4r_o$ away from its center; i.e., only including up to two hexagons formed by pipes surrounding it. This assumption simplified the calculations significantly whereas its effect upon the heat transfer rates was found to be small, as shown later.

The number of pipes at distances to $4r_o$ which affect a pipe location are shown in Table 2 for the 15 different locations which exist in the 24 x 24 matrix.

For heat sources at distances r_i, $2r_o$, $2\sqrt{3}\,r_o$ and $4r_o$, the values of $I(X_n)$ were calculated by linear interpolation of tables by Ingersoll and Plass [32] for the three time periods, 1-week, 2-week and 3-week, each with its spacing of 25, 30 or 40 inches, respectively, as shown in Table 3.

From these values of $I(X_n)$ the value of the heat transfer rate for each location was calculated from Equation (5), as shown in Table 4. It may be noted from this table that the weighted average heat transfer rate for all pipes in the 24 x 24 matrix is 69.8 Btu/hr-ft which is approximately 4% above the value of 67.1 Btu/hr-ft for a pipe at location 9, where it is completely surrounded by pipes on all sides at distances to $4r_o$, for the matrix designed for 1-week use. This improvement due to including the effect of peripheral pipes through weighted averaging is rather small considering the extent of uncertainty in the thermal diffusivity of the soil. Similar improvements of about 4% also may be noted in the calculations fo the 2-week and 3-week matrix.

The effect of surrounding pipes in the matrix on the heat transfer rate of a pipe may now be evaluated. Since about 70% of the pipes are surrounded by pipes on all sides at distances to $4r_o$, similar to location 9, the effects of the presence of pipes within distances of $2r_o$ and between $2r_o$ and $4r_o$ on a pipe will be considered separately for this location for comparison purposes. The value of $I(X_n)$ for a pipe located in an infinite earth medium in the 1-week matrix is 3.778, Table 3. If the six surrounding pipes at a distance of $2r_o$ are considered, their effect is to add 6 x 0.397 or 2.382 to the $I(X_n)$ value of 3.778. Similarly, the 12 additional pipes at distances of over $2r_o$, but not greater than $4r_o$ from location 9 add 0.9552 to the value of $I(X_n)$. From Equation (5), therefore, the heat transfer rate for the pipe located in an infinite earth medium is 126.4 Btu/hr-ft whereas the presence of pipes at a distance of $2r_o$ reduces it to 77.5 Btu/hr-ft, a reduction of 39%. Pipes located at distances over $2r_o$, but up to $4r_o$ reduce this value further to 67.1 Btu/hr-ft, an additional reduction of about 13%.

To determine the extent of errors introduced in these calculations because of not including the effects of pipes at distances greater than $4r_o$, a sample calculation was carried out for a pipe in location 16, for which the effect of an additional 18 pipe within a radius of $6r_o$

could be included, and its heat transfer rate compared with that for a pipe in location 9 calculated earlier. For the 1-week duration matrix, the heat transfer rate for location 16 was found to be 65.3 Btu/hr-ft, a decrease of 2.7% from the value of 67.1 Btu/hr-ft for location 9. It may, therefore, be seen that placement of pipes in a hexagonal arrangement at distances of $2r_o$, $4r_o$ and $6r_o$, as shown in Figure 2, causes successive reductions in the heat transfer rate of a pipe by 39%, 13% and 2.7%, respectively. The presence of additonal pipes at distances over $6r_o$ will have an effect even smaller than the 2.7% caused by 18 pipes at distances over $4r_o$ but up to and including $6r_o$. It may, therefore, be reasonable to assume that if the effect of all pipes upon all other pipes in the matrix were included, the additional deterioration in the weighted average heat transfer rate, Table 4, due to pipes at distances greater than $4r_o$ may not be greater than the 4% improvement in it caused by peripheral pipe locations. The average heat transfer for the matrix may, therefore, be assumed to be approximately that of the pipe in location 9 when the effect of pipes at distances up to $4r_o$ is included. Therefore, in determining the length of pipes required for the heat sink, values of Q' for the 1-week, 2-week and 3-week durations will be assumed to be 67.1, 53.6 and 57.5 Btu/hr-ft, respectively. The actual average heat transfer rates will be somewhat larger because of the end effects and because the entire pipe matrix is not subjected to the maximum 212°F over the total time periods of one, two and three weeks.

Dimensions of the Earth Heat Sink

With a knowledge of the average heat transfer rate from pipes in the matrix at the end of specified period of time, the length of the heat sink can be determined from the value of the total heat that has to be dissipated per hour of engine operation. The excess heat generated during a closed-cycle operation of a Diesel engine is approximately 11,500 Btu/hr per kilowatt of output in the 100 to 500 kw range of engines. For the preliminary calculations, an engine in the midrange of 300 kw may be considered for durations of one, two and three weeks. The heat to be dissipated is, therefore, at the approximate rate of 3.45×10^6 Btu/hr. From values of Q' of 67.1, 53.6 and 57.5 Btu/hr-ft obtained earlier for an assumed 24 x 24 pipe matrix for one, two and three weeks of operation, the lengths of pipes required are 51,400, 64,300 and 60,000 feet, respectively. The overall dimensions of suitable earth heat sinks are given in Table 5.

Although the overall volume of earth required to absorb the waste heat is large, the costs associated with an earth heat sink are limited to cost of excavation and select backfill which is estimated at around 50¢ per cubic foot [42] , and the cost of pipes in the matrix. Furthermore, the pipe lengths and earth volumes calculated above were not optimized for the lowest cost. An optimization involving tradeoff between the cost of pipes and the cost excavation and backfill may reduce the total cost of the earth heat sink.

For example, the values of thermal conductivity, specific heat and soil density assumed in above calculations were conservative. If care is exercised in selecting a soil of high thermal conductivity (e.g., 1.2 Btu/hr-ft-^{o}F), high specific heat (e.g., 0.45 Btu/lb-^{o}F) and a density of 100 lb/cu ft, then the value of Q' for location 9 in the matrix using the same size of pipes and same spacing between pipes will be approximately 127.6, 104.0 and 111.6 Btu/hr-ft for operations of one, two and three weeks, respectively, as shown in Table 6. The required length of pipes may then be reduced to 27,000, 33,200, and 30,900 feet for operation over one, two, and three weeks, respectively, and the overall dimensions of the heat sink reduced to those shown in Table 7.

As may be seen, the overall volume of the carefully selected soil is approximately half that of the soil with conservative thermal conductivity and specific heat values under identical operating conditions, pipe size and pipe spacing. If the spacing between pipes were increased because in a highly conducting soil the heat can diffuse further during the same time period, a saving in the overall lengths of the pipes of approximately 5000, 8700, and 6400 feet for durations of one, two, and three weeks, respectively, may be obtained at the expense of soil volume, as shown in Tables 8 and 9.

Comparing the heat sinks with alternative spacings, Tables 7 and 9, the increase in the required soil volume is 20,000, 74,000,and 90,000 cubic feet, which is approximately equivalent to 4.0, 8.5, and 14.0 cubic feet of soil per foot of pipe length saved for the 1-week, 2-week, and 3-week durations, respectively. At an estimated cost of 50¢ per cubic foot of pipe for excavation and backfill, the alternative heat sink of Table 9 for the 1-week service will be more economical if the cost of the heat exchanger in the matrix is equivalent to less than $2.0 per foot of pipe used in its construction.

DISCUSSION

The cost of an earth heat sink may now be estimated to provide a basis for comparison with other heat dissipation methods. A heat sink matrix made from suitable stainless steel pipes of 1.5-inch diameter which can withstand external pressures of about 1000 psi may be assumed to cost approximately $2.0 per foot length of pipe. Similarly, a cost of $3.00 per foot of pipe may be assumed for service at overpressures of about 1500 psi. The cost of excavation and select backfill may be assumed to be approximately 50¢ per cubic foot. For the purpose of this estimate, the costs associated with hardening the heat sink against ground motion will not be included. Using values developed in Table 7, the cost of the earth heat sink for 1-week service is $109,000; the cost of pipes and the cost of excavation and select backfill being approximately equal. The total heat dissipated by this heat sink over a period of one week will be 5.79 x 10^8 Btu, or approximately 5300 Btu per dollar cost of the earth heat sink. Similarly, the heat dissipated per dollar cost of the earth heat sink for the 2-week and 3-week duration eart heat sinks is 7200 Btu and 7700 Btu, respectively. Table 10 details results of calculations for the two types of soil considered here. With optimization involving pipe size, pipe spacing and the material for select backfill, it should not be difficult to increase heat dissipation per dollar cost above those shown in Table 10. Using select backfill, the heat dissipation per dollar cost of between 5300 and 7700 Btu appears to be relatively inexpensive.

Looking at it from another viewpoint, calculations were carried out to determine the Btu absorption capacity of the earth heat sink in Btu/cu ft, as shown in Table 11. If water at 60°F or ice at 32°F stored underground was used, their heat absorption capacities will be approximately 7,200 Btu and 16,400 Btu per cubic foot, respectively. Actual volumetric advantages of using water or ice will be somewhat less than that shown by a comparison of these figures with those of Table 11 since the volume of tank to hold water or ice, volume of insulation surrounding the ice tank, the volume of refrigeration equipments needed to produce and maintain ice and the volume of heat exchangers needed to affect the heat transfer were not included in these estimates.

Meaningful cost comparisons with alternative methods of heat dissipation require a knowledge of overpressures, duration of use, cost of protecting the earth heat sink against ground motion, and the cost of alternative methods of heat dissipation and their development cost. Most of this information is not available at this time. Comparison of earth heat sinks with other heat dissipation methods are, however, now being carried out for conditions applicable to project SANGUINE. Preliminary comparisons are encouraging.

CONCLUSIONS

Preliminary calculations have shown that earth heat sinks may be practical for dissipation of waste heat from underground power plants. Preliminary calculations show that an earth heat sink using select backfill may dissipate approximately 6000 to 8000 Btu per $ cost and 5000 to 6000 Btu per cubic foot volume of the earth heat sink. Optimization of design may produce further improvement in performance of the earth heat sinks.

RECOMMENDATIONS

1. A complete solution should be obtained for the case of a long, concentric circular cylinder whose ends and outer wall are perfectly insulated and whose inner wall is raised and maintained at a fixed temperature above the uniform, initial cylinder temperature. The solution should be plotted in graphical form for values of the soil properties, the inner and outer diameters and the operating period varying over wide ranges.

2. Using the results of the above analysis, an optimization of earth heat sinks should be carried out to obtain the lowest overall volume and the lowest overall cost.

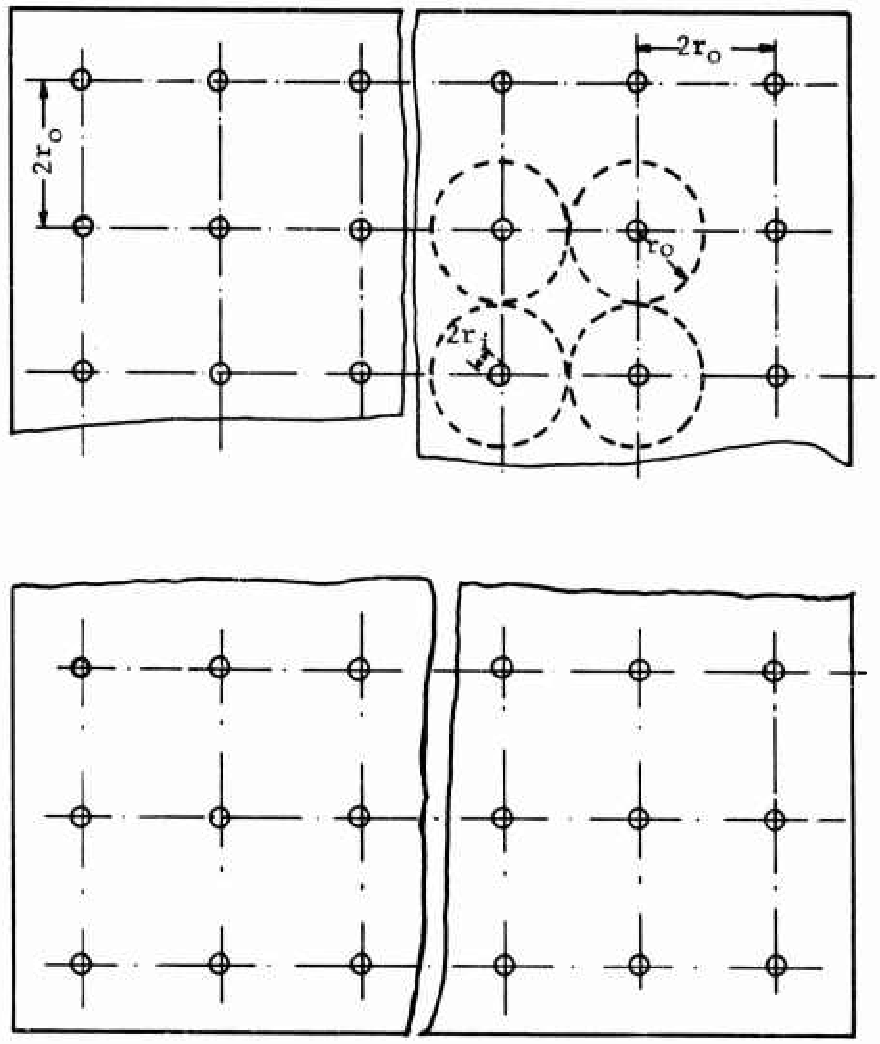

Figure 1. Cross-section of square prism matrix of pipes in an earth heat sink.

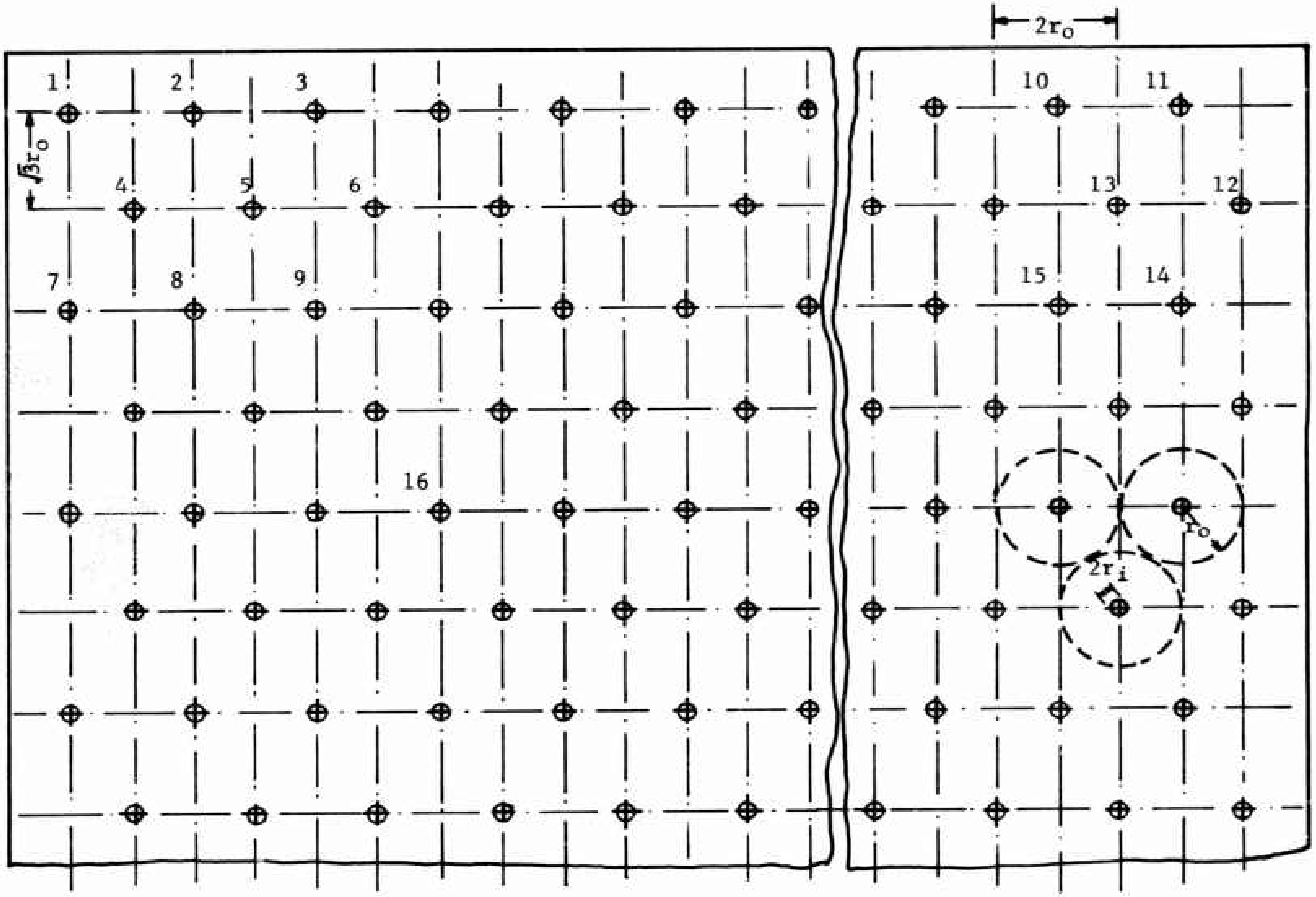

Figure 2. Cross-section of hexagonal prism matrix of pipes in an earth heat sink.

Table 1. Thermal Conductivities of Two Soil Samples at 60°F, [28].

Sample	Density lb/cu ft	Moisture %	k Btu/hr-ft-°F
Sandy-type soil	82	9.9	0.6
	86	4.7	0.3
	91	9.9	0.7
	95.6	4.7	0.45
	100	9.9	1.0
	103	11.9	1.25
	105	4.7	0.6
	112	11.9	1.47
Clay and Loom-type soil	48	41.4	0.33
	51	57	0.43
	52	29.7	0.28
	53	41.3	0.37
	57	57	0.60
	59	29.7	0.33
	60	41.3	0.42

Table 2. Location of various pipes in a 24 x 24 hexagonal prism matrix.

Location of pipes (Fig. 2)	No. of pipes at similar locations	Number of pipes at a distance of		
		$2r_o$	$2\sqrt{3}r_o$	$4r_o$
1	2	2	2	2
2	2	4	2	3
3	40	4	3	4
4	2	5	3	2
5	2	6	5	3
6	40	6	5	4
7	20	3	3	3
8	20	6	4	4
9	400	6	6	6
10	2	4	3	3
11	2	3	2	2
12	2	3	3	2
13	2	6	4	3
14	20	5	4	3
15	20	6	6	5

Table 3. Values of $I(X_n)$ for pipes in a 24 x 24 hexagonal prism matrix in an earth heat sink of α = 0.0185 sq ft/hr.

Time (hr)	Pipe Spacing (in)	$2\sqrt{\alpha t}$ (ft)	r (ft)	X_n -	$I(X_n)$ -
168	25	3.52	0.0625	0.0178	3.7780
			2.083	0.5918	0.3970
			3.610	1.0256	0.1007
			4.167	1.1837	0.0585
336	30	4.98	0.0625	0.0126	4.1390
			2.50	0.5020	0.5191
			4.33	0.8701	0.1680
			5.00	1.0040	0.1083
504	40	6.10	0.0625	0.0103	4.2990
			3.333	0.5464	0.4548
			5.770	0.9459	0.1313
			6.667	1.0929	0.0810

Table 4. Heat transfer rates using $I(X_n)$ values from Table 3 for pipe locations shown in Figure 2 in an earth heat sink of α = 0.0185 sq ft/hr, and k = 0.5 Btu/hr-ft-°F

Location of pipe in the matrix	$\Sigma I(X_n)$			Q' (Btu/hr-ft)*		
	1-week	2-week	3-week	1-week	2-week	3-week
1	4.8904	5.7298	6.5332	97.6	83.3	84.8
2	5.7429	6.8763	6.6238	83.1	69.4	72.1
3	5.9021	7.1526	6.8361	80.9	66.8	69.9
4	6.1821	7.4551	7.1289	77.2	64.1	67.0
5	6.8390	8.4185	7.9273	69.8	56.7	60.2
6	6.8975	8.5268	8.0083	69.2	56.0	59.6
7	5.4466	6.5282	6.3003	87.7	73.2	75.8
8	6.7968	8.3588	7.8770	70.3	57.1	60.6
9	7.1152	8.9114	8.3016	67.1	53.6	57.5
10	5.8436	7.0443	6.7551	81.7	67.8	70.7
11	5.2874	6.2489	6.0880	90.3	76.4	78.4
12	5.3881	6.4169	6.2193	88.6	74.4	76.8
13	6.7383	8.2505	7.7960	70.9	57.9	61.3
14	6.3413	7.7314	7.3412	75.3	61.8	65.0
15	7.0567	8.8031	8.2206	67.7	54.2	58.1
Weighted average for the 24 x 24 matrix-------------------------				69.8	56.2	59.9

* see spacings between pipes in Table 3.

Table 5. Dimensions of earth heat sinks of α = 0.0185 sq ft/hr and k = 0.5 Btu/hr-ft-°F for a 300 kw Diesel using a 24 x 24 hexagonal prism pipe matrix.

Duration of Operation (hr)	Pipe dia. (in)	Pipe Spacing (in)	Dimensions of the Heat Sink			
			Length (ft)	Width (ft)	Depth (ft)	Volume (cu ft)
168	1.5	25	91	51	44	204000
336	1.5	30	114	61	52	362000
504	1.5	40	106	82	70	608000

Table 6. Heat transfer rates for a pipe in location 9 in the Figure 2 matrix in an earth heat sink of α = 0.0267 sq ft/hr and k = 1.2 Btu/hr-ft-°F.

Time (hr)	Pipe Spacing (in)	$2\sqrt{\alpha t}$ (ft)	r (ft)	X_n -	$I(X_n)$ -	$\Sigma I(X_n)$ for location 9	Q' (Btu/hr-ft)
168	25	4.2332	0.0625	0.0148	3.9867	8.983	127.6
			2.083	0.4921	0.5395		
			3.610	0.8528	0.1776		
			4.167	0.9844	0.1156		
336	30	5.9867	0.0625	0.0104	4.2861	11.021	104.0
			2.500	0.4176	0.6683		
			4.333	0.7233	0.2663		
			5.000	0.8352	0.1878		
504	40	7.3320	0.0625	0.0085	4.4785	10.267	111.6
			3.333	0.4546	0.5982		
			5.765	0.7870	0.2186		
			6.667	0.9093	0.1480		

Table 7. Dimensions of earth heat sinks of α = 0.0267 sq ft/hr and k = 1.2 Btu/hr-ft-°F for a 300 kw Diesel using a 24 x 24 pipe matrix.

Duration (hr)	Pipe dia. (in)	Pipe Spacing (in)	Dimensions of the heat sink			
			Length (ft)	Width (ft)	Depth (ft)	Volume cu ft
168	1.5	25	49	51	44	110000
336	1.5	30	60	61	52	190000
504	1.5	40	57	82	70	327000

Table 8. Heat transfer rates for a pipe in location 9 in the Figure 2 matrix in an earth heat sink of α = 0.0267 sq ft/hr and k = 1.2 Btu/hr-ft-^{o}F.

Time (hr)	Pipe Spacing (in)	$2\sqrt{\alpha t}$ (ft)	r (ft)	$\underline{X}_n$	$I(\underline{X}_n)$	$\Sigma I(X_n)$ for location 9	Q' (Btu/hr-ft)
168	30	4.2332	0.0625	0.0148	3.9867	7.3437	156.0
			2.500	0.5906	0.3984		
			4.325	1.0217	0.1020		
			5.000	1.1811	0.0591		
336	40	5.9867	0.0625	0.0104	4.2861	8.1249	141.0
			3.333	0.5567	0.4409		
			5.765	0.9630	0.1241		
			6.667	1.1136	0.0748		
504	50	7.3320	0.0625	0.0085	4.4785	8.1469	140.7
			4.167	0.5683	0.4260		
			7.210	0.9834	0.1160		
			8.333	1.1366	0.0694		

Table 9. Dimensions of earth heat sinks of α = 0.0267 sq ft/hr, and k = 1.2 Btu/hr-ft-^{o}F for a 300 kw Diesel using a 24 x 24 pipe matrix.

Duration (hr)	Pipe dia. (in)	Pipe Spacing (in)	Dimensions of the heat sink			
			Length (ft)	Width (ft)	Depth (ft)	Volume cu ft
168	1.5	30	41	61	50	130000
336	1.5	40	46	82	70	264000
504	1.5	50	47	102	87	417000

Table 10. Cost of earth heat sinks.

For service at overpressures to 1000 psi								
Duration of use	Cost of pipes K$		Cost of excavation and backfill K$		Total Cost K$		Btu/$	
(hrs)	A*	B*	A	B	A	B	A	B
168	103	54	102	55	205	109	2800	5300
336	129	67	181	95	310	162	3700	7200
504	120	62	304	164	424	226	4100	7700
For service at overpressures to 1500 psi								
168	154	81	102	55	256	136	2300	4200
336	193	100	181	95	374	195	3100	5900
504	180	93	304	164	484	257	3600	6800

*A = Soil of k = 0.5 Btu/hr-ft-^{o}F, $\alpha = 0.0185$ sq ft/hr, Table 5.

*B = Soil of k = 1.2 Btu/hr-ft-^{o}F, $\alpha = 0.0267$ sq ft/hr, Table 7.

Table 11. Heat absorption capacity in Btu/cu ft.

Duration of use (hr)	Heat to be absorbed (Btu)	Volume of heat sink, k cu ft		Absorption capacity Btu/cu ft	
		A*	B*	A	B
168	5.706×10^8	204	110	2840	5270
336	1.159×10^9	362	190	3200	6100
504	1.739×10^9	608	327	2860	5300

* A = soil of k = 0.5 Btu/hr-ft-°F, α = 0.0185 sq ft/hr

* B = soil of k = 1.2 Btu/hr-ft-°F, α = 0.0267 sq ft/hr

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NOMENCLATURE

C_p	Specific heat of soil	Btu/lb-°F
J_o, J_1	Bessel functions of the first kind	-
k	Thermal Conductivity of soil	Btu/hr-ft-°F
n	Number of pipes in the array	-
N_f	Fourier's number, $r^2/\alpha t$	-
p	The ratio of distance of a location under consideration from the center of the pipe to the radius of the pipe.	-
Q'	Heat transfer rate per foot of pipe	Btu/hr-ft
r	Distance from center of pipe	ft
r_1	Radius of the pipe	ft
r_2, r_3, etc	Distance of heat sources or sinks from the center of pipe of radius r_1	ft
r_i	Inner radius of a cylinder	ft
r_o	Outer radius of a cylinder	ft
T	Temperature at distance r at time t	°F
T_1	Temperature of pipe of radius r_1 at time t	°F
T_o	Equilibrium earth temperature at time zero	°F
t	Time	hr
X	$r/2\sqrt{\alpha t}$	-
Y_o, Y_1	Bessel function of the second kind	-

Z	$\alpha t/r^2$, or $1/N_f$	-
α	Thermal diffusivity, $\frac{k}{\rho C_p}$	sq ft/hr
β	Variable of integration	-
π	3.14159	-
ρ	Density of soil	lb/cu ft